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Variation with height of rainfall below the melting level.

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AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

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Summary

3-cm radar intensity measurements, made at a range of heights up to 6,000 ft on three days with widespread moderate rain, suggest that there was considerable uniformity of drop-size distribution with height once melting of snowflakes was complete and terminal velocity was achieved, and that all significant growth of precipitation was taking place at and above the melting level.

The measurements suggest that the cloud-water content below the melting level was low, and this may prove to be typical of warm-front precipitation in those parts of the British Isles in which orographic effects are unimportant.

The contribution to signal intensity due to the coalescence of drops (as distinct from the coalescence of droplets as a growth-initiation mechanism), which has been computed by Mason and Ramanadham (1) to be 0.8 db/km for closely similar conditions to ours, could not be detected.

Rates of rainfall calculated from the radar equation agree reasonably well with measured values if a factor F of value 0.15 is inserted in the equation. This is in close agreement with the value 0.2 obtained in America by Austin and Williams (2), who have made a particularly extensive series of measurements using pulse integration methods.

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1. Introduction

It must be admitted that the quantitative use of radar in the study of rainfall has made relatively little progress. There has for example been doubt as to whether radar signals which are received from precipitation above the ground can be immediately related to rainfall measurements or drop-size distributions taken at ground level.

Mason and Ramanadham (1) have made a positive contribution to this by making computations of accretion effects and collision processes below the melting level. They found that after a fall of only 1 km in layer cloud conditions the smaller drops should be seriously depleted and the larger ones increased.

Confirmation of these effects is desirable but is hard to come by. Reliable measurement of raindrops from aircraft has not yet been achieved. Balloon-borne measuring instruments have been developed in Australia (Adderley (3)) but are attended with great difficulty and have given some unexplained results. In radar however we have a method of measurement which is particularly sensitive to changes in drop-size distribution. Radar echo intensity from rain is proportional to $\sum d^6$, where d is the drop diameter, the summation being taken over all the drops in unit volume. It is evident that

small drops and cloud droplets, although occurring in larger numbers than big drops, provide a negligible contribution to the received echo, and that the echo intensity will attain large values when large drops are present.

Mason and Ramanadham also computed theoretical radar echo intensitites, and concluded that growth of drops by collision and accretion should, in the steady rainfall conditions they assumed, be detectable by radar if the fall path is greater than about 1 km. The present paper reports measurements made in such conditions.

2. Radar measurements

There is considerable advantage to radar in studying 'steady state' rain in widespread rain conditions. There is then no doubt that the radar beam is being filled by the precipitation, and it is advantageous to scan in elevation at a fixed range and azimuth which have previously been checked as being free from unwanted permanent echoes from ground targets. Wexler and Austin (4) and Boucher et al (5) have made intensity studies of this type, but have concentrated their attention on the variations of intensity in the melting level and in the snow above, and have not reported any detailed measurements below the melting level. Their working range was 11 mi, and they scanned in steps of 1 or 2^{o} in elevation, i.e. steps of 1,000 or 2,000 ft in height.

It might be thought that use of a vertically-looking radar would be preferable to scanning in elevation because of the short ranges which would then be involved. A great disadvantage of this method is that, when a single aerial has to be used for both transmitting and receiving, the receiver suffers from paralysis for the duration of the transmitted pulse, and with our equipment no signals are received from the lowest 2,000 ft. It was therefore decided to scan in elevation at a range of about 3 mi. This made it possible to study in detail the lowest 6,000 ft in steps of 250 or 500 ft. Reliable measurements could not be made below 500 ft because of obstructions in the path of the beam. Results obtained on three days with widespread continuous moderate rain are presented. On each occasion there is independent radar evidence that the rain was reasonably uniform over an area of several hundred square miles.

3. Radar theory

The signals from a calibrated radar can be interpreted in terms of rate of rainfall if drop-size distribution is known. For precipitation uniformly filling the beam at range r the received power averaged over many pulses P_r is given by

$$P_r = \frac{P_o h A_e}{8 \pi r^2} \Sigma\sigma \qquad \ldots\ldots\ldots (1)$$

P_o	peak output power
h	pulse length
A_e	effective area of antenna
$\Sigma\sigma$	scattering cross-section of the precipitation per unit volume.

A_e usually has a value between 0.5 and 0.85 of the actual area A of the antenna, and is constant for any one antenna. The reduction factor is caused by the non-uniformity of the radiation across the transmitted beam.

The scattering cross-section of a single drop which is small in relation to the wavelength has been evaluated by Ryde (6) as

$$\sigma = 4\pi a^2 \left(\frac{2\pi a}{\lambda}\right)^4 \left|\frac{\epsilon - 1}{\epsilon + 2}\right|^2 \qquad \ldots\ldots\ldots (2)$$

a	radius of drop
λ	wavelength of radar
ϵ	dielectric constant

/The

The dependence of σ on λ is seen to be a form of Rayleigh scattering. (2) simplifies to

$$\sigma = \frac{\pi^5 d^6 |K|^2}{\lambda^4} \qquad \ldots\ldots\ldots(3)$$

where $K = \epsilon - 1/\epsilon + 2$, d diameter of drop.
Inserting in (1) we have

$$P_r = \frac{P_0 h A_e \pi^4 |K|^2 Z}{8 \lambda^4 r^2} \qquad \ldots\ldots\ldots(4)$$

where $$Z = \sum d^6 \qquad \ldots\ldots\ldots(5)$$

the summation being taken over all the drops in unit volume.

Numerous studies of drop-size distribution at the ground have been made, both in Europe and America. A valuable summary of this work was undertaken by Best (7). He found a relationship of the form $Z = a R^b$, where R is rate of rainfall and a and b are constants. a and b showed some variability, but mean values gave

$$Z = 346\ R^{1.54} \qquad (Z \text{ mm}^6\text{m}^{-3},\ R \text{ mm hr}^{-1}).$$

Inserting in (4) we have

$$P_r = 4{\cdot}33 \times 10^{-11} \pi^4 P_0 h A_e |K|^2 \lambda^{-4} R^{1.54} r^{-2} \qquad \ldots\ldots\ldots(6)$$

(R mm hr^{-1}, all other quantities in cgs units).

This is a very convenient form for interpretation of radar echo intensities in terms of rate of rainfall.

4. Scanning technique

The radar measurements were made on the amplitude display of the range-height radar AN/TPS-10. This is American equipment, designed to scan in elevation from 0 to 22° on any chosen bearing. The beam width (to half-power points) of 0.7° in the vertical samples a layer 200 ft deep at the selected working range of 15,000 ft, while its 2° beam in azimuth is 500 ft wide at this range. The signal appearing at any point in the display is that from a range interval equal to half the pulse length, in this case 500 ft. The measurements were thus made along a column 500 ft square in cross-section. The use of a constant slant range meant that the highest measurements were on a sample displaced 1,200 ft from the lowest sample. This slanting column is quite acceptable for steady rain scans, and makes range correction of individual readings unnecessary. In any case the wind drift of precipitation through the column in the 2 to 5 min taken to complete a scan is usually much more than 1,200 ft, and cannot readily be allowed for.

The effect of attenuation of the 3 cm beam by the intervening rain can be safely neglected for this short double path-length of 6 mi and the steady rain intensities involved. It would amount to less than 1 db, and being very closely the same for the constant slant range and the scanning elevation involved, would not affect comparisons in elevation.

/5.

5. Absolute intensity measurements

Radar signals can be compared by direct measurement on an amplitude display, but they cannot in this way be related to an absolute level. A signal generator test set TS-263 was therefore used to calibrate the gain dial of the radar in decibels (db) before and after each set of intensity measurements, and this calibration was normally found to be stable within 1 db. To convert these signal generator db values to dbm (decibels below the level of 1 milliwatt), a factor of +45 db is applied. The various contributions to this factor are listed in Table 1.

Table 1. Factors to be applied to signal generator readings to relate to level below 1 milliwatt

Cause	Factor db	Notes
Insertion loss of directional coupler	+37	Makers' calibration 37 db $\pm\frac{1}{2}$ db.
Loss in concentric cable between directional coupler and signal generator	+2	Makers' calibration
Attenuation in waveguide between horn and directional coupler	-1	Affects received signals only. TPS-10 handbook
Difference in saturation levels of the displays used for calibration and for precipitation scanning	+2	Measured on gain dial
Difference between signal generator pulse level and precipitation setting level	+5	See footnote *
Total factor	+45	

The following quantities are known or can be measured:-

P_o peak power output measured on test set

h pulse length 1 μ sec (3×10^4 cm)

$|K|^2$ 0.93 for water, practically independent of temperature

A_e effective area of antenna. This is obtained from the known aerial gain G, 18,000. The gain of a given aerial over an isotropic radiator is given by $G = \frac{4\pi A_e}{\lambda^2}$ from which we obtain A_e 15,600 cm^2.

Footnote * This combines two corrections. Precipitation echoes, which result from incoherent scattering, give a rapidly beating signal of the same nature as receiver noise. It is surprisingly difficult to bring the peaks of a beating signal accurately up to saturation level on an amplitude display. It proved more satisfactory to make measurements by increasing the gain until the 15,000 ft range marker shape could just be detected at the saturation level. This required a slightly higher gain setting than for measurement on the peaks.

The peak amplitude of a randomly beating signal is greater than its rms value. From gain dial settings in steady precipitation it was concluded that the range marker level was nearly twice the rms amplitude i.e. peak intensity between 3 and 4 times the rms intensity.

The signal generator output on the other hand, although pulsed, is long compared with precipitation echo peaks, and it is more appropriate to compare this with the rms value of a beating echo. This factor 3 to 4 leads to the correction 5 db.

We can now interpret variations of signal intensity with height in equation (6) in terms of rate of rainfall, for comparison with raingauge values. The nearest recording raingauge was at Dunstable, 4 mi west of the scanning site.

6. 5th September 1955

Continuous moderate rain associated with a well-defined trough fell at East Hill from 0830 to 0945 GMT. The 10 cm plan-position photograph (Fig.2) taken at 0929 shows a belt of precipitation 20 to 30 mi wide lying over the station. The rainfall recorder at Dunstable, which is only 2 mi south-west of East Hill (though 4 mi from the scanning site) shows particularly steady rain from 0910 to 0940 of 2.7 mm hr^{-1}. In all 7.4 mm fell in the period 0800 to 1400.

The 10 cm range-height photograph (Fig.1) taken at 0845 looking along the belt of precipitation shows echo to 20,000 ft. The steadily falling echo top out to a range of 35 mi is an effect of the increasing range, but its evenness is a clear indication of the steadiness of the precipitation. As uniform an echo as this is quite unusual. A further reliable guide to the steadiness of the precipitation is the reduced gain photograph (Fig.3) taken at 0943 using an extended range scale. This shows an unusually uniform echo in the melting level at about 8,500 ft. A full description of the bright band at the melting level has been given by Browne, Palmer and Wormell (8) and will not be repeated here. It will be noticed that there is practically no echo within 5 mi of the station in Fig.3. This is not due to weaker rain, but is a 'swept gain' effect built into the receiver where strong echoes from ground targets are a permanent feature of the display with the wider angle beams of the 10 cm equipment (it is only by careful selection of azimuth and because of the much narrower beam of the 3 cm radar that these effects can be avoided when scanning at a range of 3 mi).

The normal range photograph (Fig.4) taken at 0945 shows that a cellular structure had developed beyond 25 mi by 0945, with almost vertical columns of heavier precipitation imbedded in lighter precipitation. It must be remembered that the height scale on this display is nearly five times as open as the range scale so that an echo column which appears twice as tall as it is wide is actually coming from a precipitation column twice as wide as it is tall.

Four complete scans were made with the 3 cm range-height radar in the period 0916 to 0932, with measurements at each degree step in elevation. Each scan took about four minutes. The results plotted as signal intensity against height (Fig.5) show that rainfall was heaviest at 0916. The unsteady and generally low signal intensities of the scan made at 1000 were in much weaker rainfall, and operations had to be discontinued. In fact rain ceased at East Hill at 1003 and at Dunstable at 1000.

A logarithmic scale of rate of rainfall has been added from equation (6) based on the rate of fall of 2.7 mm hr^{-1} at Dunstable. This scale has been added to give more meaning to the signal intensity values. It should not be regarded as a strictly absolute scale, though internal evidence for its approximate validity will be brought forward in para.9.

The top of the scan at 5,600 ft was well below the melting level, and it is clear that the radar measurements were not affected by the melting process. Discussion of the results is reserved for para.11.

7. 19th October 1955

A deep and slow-moving depression centred over Ireland (pressure at centre 976 mb) gave 21.1 mm of rain at Dunstable in the 24 hours ending 0900/20 but the continuous moderate to heavy rain from 1010 to 1150 on the 19th, when 8 mm of this fell, seems to have been associated with the rapid eastward movement across Southern England of a narrow warm sector. The plan-position photograph (Fig.8) taken at 1020 in the middle of the intensity measurements shows extremely uniform echo extending between 20 and 50 mi in all directions (the lack of brightness in this picture is not caused by weakness of echo but by a slight photographic fault).

The extended scale range-height photographs taken at 1005 (Figs.6 and 7) show uniform echo out to 20 mi and an extremely uniform bright band at the melting level at 5,000 ft. There is an indication of stronger precipitation at $22\frac{1}{2}$ mi, and this is again shown in the normal range photograph taken at 1021 (Fig.9).

Thirteen scans were made with the 3 cm range-height radar in the period 0955 to 1025 GMT. This was only possible by taking intensity measurements in 4 deg. steps in elevation, each scan then taking about 2 min. The scans are numbered consecutively on the signal intensity-height plots (Fig.10) which for clarity are plotted on two graphs. The rain was clearly very variable for scans 1 and 2. Scan 3 at 1005 is much steadier, but shows no sign of the strong echoes to be expected from the melting band, the base of which must have been above 5,600 ft at this time.

Scans 4 to 11 between 1010 and 1022 show a very definite melting band effect at 5,600 ft and relatively uniform intensity below 5,000 ft. The scale of rate of rainfall is based on the 3.5 mm hr^{-1} recorded at Dunstable for this period.

Scan 13 at 1024 was in heavier precipitation, but both 12 and 13 seem to indicate a further slight lowering of the base of the melting layer. This may however have been due to a deepening of the layer by cooling by the melting snow rather than to a lowered freezing level. For discussion of the intensity measurements see para.10.

8. 20th October 1955

By 1200 GMT 20th October the depression which was over Ireland on the 19th had filled to 984 mb and was moving up the Norwegian coast. A cold northerly airstream had spread to most parts of the British Isles, but a wave had developed on the cold front of the depression, and a period of continuous rain was experienced in the Dunstable area during its passage. The Dunstable rain recorder collected 5 mm of rain in the 24 hours ending 0900/21, and 4 mm of this fell in the period 0900 to 1230 on the 20th. Measurements were made in two periods and it is convenient to treat them separately.

(a) 0955-1025 GMT

The 10 cm plan-position photograph taken at 1015 (Fig.13) shows clearly the uniform echo from this rain area. It extends to ranges of 30 to 60 mi from East Hill. The range-height photographs at 0932 and 1035 (Figs.11 and 14) show the echo tops as 20,000 ft close to the station, while the reduced gain picture at 0933 (Fig.12) again shows a uniform bright band as on 19th but now down to 4,000 ft.

The first 3 scans in the period 0955 to 1005 were made only on the rain below the bright band in 2 deg steps from 16 down to 2 deg. As the rain showed no sign of abating, scans 4 to 9 investigated in addition the bright band in some detail, in 1 deg steps from 22 to 14 deg and then 2 deg steps as before. The results, plotted in Fig.15, are of particular interest. The top of the melting level is particularly well-defined in all the scans, and shows remarkably little variation in height. The melting process and the acceleration of the resulting raindrops to terminal velocity appears to be complete in a layer only a little over 1,000 ft deep, for below this there is relatively little change in intensity, and hence in rate of rainfall, down to ground level.

There would seem to be definite evidence of a deepening of the melting layer by lowering of its base by perhaps 500 ft in 15 min in this series of observations. This is probably due to cooling by the melting snow, but advection of colder air may be a contributory cause.

The rain was clearly heaviest at the beginning and end of the period. It was steady throughout scans 4 to 7 (1014 to 1017) but the logarithmic scale of rate of rainfall is based on the average rate of fall at Dunstable for the 8 scans in the period 1158 to 1228 GMT.

/(b)

(b) 1158-1228 GMT

Dunstable recorded steady rainfall of 1.2 mm hr^{-1} in this period, and a further series of 8 scans was made at the same range 15,000 ft and with the same technique i.e. 1 deg steps through the bright band and 2 deg steps beneath.

The short range plan-position photograph (Fig.16) taken at 1156 shows the uniform precipitation echo on which the measurements were made. It is especially uniform from SE to SW of the station. The medium range picture taken immediately after completion of the series at 1230 (Fig.18) shows the same rain area almost past East Hill, but still uniform to the SE, the direction in which the scans were made. The 10 cm range-height photograph taken at 1212 (Fig.17) shows uniform echo in the first 20 mi with echo top 11,000 ft close to East Hill. The small patch of echo at 15,000 ft at range 13 mi is evidence however that there was still some precipitation at higher levels but that it was not as widespread or intense as it had been two hours earlier. The final range-height photograph (Fig.19) was taken at 1246 looking northeast through the rain, which had by then cleared East Hill.

The results are again plotted on two intensity-height graphs (Fig.20). All but one scan again show a well-defined top to the melting layer at 5,000 ft (these heights are above station level; 400 ft must be added to reduce to msl). The reason for the lower top on scan 5 is not clear. The bottom of the bright band was exceptionally uniform for the last three scans. As for series (a) the bright band was again only a little over 1,000 ft thick.

The temperature scales which have been added at the right-hand edge of each intensity-height plot are based on the 1400 GMT ascents at Hemsby, 100 mi northeast of East Hill. It will be noticed that the maximum of intensity in the bright band at the melting level on this day lies on the average 1,500 ft below the freezing level found at Hemsby. It is usually closer to the freezing level than this, and the results found by Hooper and Kippax (9) are thought to be typical. They made radar observations of the bright band at a radiosonde station simultaneously with the radiosonde ascents, ten determinations giving a difference of 440 ft between the two levels with a probable error of $\pm$ 150 ft. The present large difference is due to a near-isothermal layer between 4,000 and 6,000 ft which can be seen reflected in the temperature scale; the layer from 0 to 2.5°C is almost 3,000 ft deep while that from 2.5 to 5°C is barely 1,000 ft deep. This layer is probably due to prolonged cooling by melting snow.

A scale of rate of rainfall has again been added to the intensity plots based on the steady fall measured at Dunstable in the period of these scans.

9. Calculated rates of rainfall

The mean value of signal intensity in the rain below the bright band in each of these series has been inserted in Equation (6) and a calculated rate of rainfall obtained. These values are given in col.4 of Table 2, and are seen to be in every case much lower than the measured rates of fall. This has been the

Table 2. Calculated and measured rates of rainfall

Date	Time GMT	No. of scans	R mm hr^{-1} Radar	R mm hr^{-1} Raingauge	F
5. 9.55	0916-0932	4	0.74	2.7	0.14
19.10.55	1010-1022	8	1.25	3.5	0.21
"	1022-1025	2	1.96	5.7	0.19
"	Max. at 1024	1	2.28	-	-
20.10.55	1006-1016	4	0.09	1.4	0.01
"	1158-1228	8	0.33	1.2	0.14
"	1220-1227	3	0.41	-	-

/experience

experience of most workers in this subject. Almost invariably a marked discrepancy has been found between theory and measurement. Austin and Williams (2) have probably made the most extended investigation, using pulse integration methods. They found it necessary to insert a factor F of value 0.2 in the numerator of equation (6) to make rainfall from radar and gauge agree i.e. the intensity of the signal received from precipitation was only one fifth that which theory indicated.

Values found for this factor by various workers are given in Table 3. The extremely high value derived from the measurements of Hooper and Kippax (9)

Table 3. Values for the factor F which have been found by various workers (measurements on rain unless otherwise stated)

	F	λ cm	Notes
Austin & Williams	0.2	10	Pulse integration methods
Hood	0.4	10	
Langille & Thain	0.6	3	
Hooper & Kippax	1.25	3	
Hood	0.1	3	
Kodaira	0.13	3	
Robinson	0.8	0.8	
Donaldson	0.1		Measurements on fog at millimetric wavelength

calls for some comment. Their measurements on rainfall (0.5 - 10 mm hr^{-1}) gave signal intensities averaging 1 db above the theoretical relationship, giving F 1.25. Mr. Hooper has confirmed however that no allowance was made for the difference in the criteria used in making amplitude measurements of the precipitation echoes and the signal generator pulse (see para.5). Robinson (10), working more recently at Malvern, and making similar amplitude measurements allowed 4.8 db for this, in close agreement with the 5 db we derived independently. 4.8 db applied to Hooper and Kippax's data would give F 0.4.

It must be remembered too that all these assessments of F depend on the drop-size relationship which has been assumed. Hooper, Kippax and Robinson, for example, accepted Ryde's theoretical relationship which was based on the drop-size distribution studies of Laws and Parsons in America. Best has shown that these can be represented by $Z = 317\ R^{1.46}$. If Best's relationship $Z = 346\ R^{1.54}$ had been assumed the values of F would have been further reduced to 0.3 (Hooper and Kippax) and 0.6 (Robinson).

A value of F has been calculated for each period of uniform rain of Table 2 for which a reliable raingauge value is available. These are given at col.6. The comparative uniformity of four out of five of the values is confirmation of the uniformity of the rain in space and time. The weighted mean of these 26 scans gives F 0.15. This is sufficiently close to the 0.2 found by Austin and Williams (2) to justify applying this value in calculation of rates of rainfall. This has been done in Table 4. The agreement is seen to be good in four cases out of five.

The necessity for introducing a factor F remains obscure. By careful calibration and the use of a standard metal sphere Austin and Williams have shown that it is not instrumental in origin, nor is it due to the non-uniform distribution of radiation across the radar beam. It would seem to be inherent in the nature of the beating signal back-scattered from precipitation. Marshall, Hitschfeld and Gunn (11) suggest that it may prove to be related to an unusual finding of Dr. W.E. Gordon of Cornell who has found in the course of radio propagation studies that the energy received is not proportional to A_e when the antenna dimensions are greater than the 'correlation distance', i.e. the distance between two points for which the received signals have zero correlation. This work is at present unpublished, and it is perhaps premature to refer to it here.

/Table 4.

Table 4. Comparison of measured and calculated rates of rainfall assuming F has the value 0.2

Date	Time GMT	R mm hr^{-1} Measured	Calculated
5. 9.55	0917-0930	2.7	2.1
19.10.55	1010-1022	3.5	3.5
"	1022-1025	5.7	5.6
20.10.55	1006-1016	1.4	0.3
"	1158-1228	1.2	0.9

10. Errors in calculating rainfall

The values of Table 4 should be considered in relation to the possible errors. Comparison of measured and calculated rainfall may be in error due to three main causes:-

(a) radar calibration errors

(b) drop-size distributions not conforming to the relationship $Z = 346\ R^{1.54}$

(c) raingauge measurements not being made on the actual echoing rain.

None of these factors can be easily assessed

(a) The signal generator calibration readings have a standard deviation of about 1 db from the mean. This is probably due to slight instability in the receiver. We can expect that the gain dial setting error is of the same order. In addition it is probable that the transmitted power cannot be measured with our present equipment to better than 10% accuracy. When these effects are combined and interpreted in terms of rate of rainfall we obtain the probable error of a single calculation of rainfall as 35%. The scatter of the points in a single scan in the nominally steady rain below the bright band is seen to be of this order.

(b) Farnsworth (12) has found that even an accurately calibrated radar can indicate rainfall rates in error by a factor of two due to variability of drop-size distribution. His measurements however included a wide range of rates of rainfall, and smaller errors may be expected when measurements are limited to conditions of continuous moderate rain from layer cloud.

(c) It is clearly desirable in future measurements to scan directly above a raingauge or raingauge network.

11. Discussion of results

If the 3 scans made in extremely unsteady rain are excluded we can find 5 of the remaining 32 scans which show a slight tendency to increase of signal intensity towards the ground, averaging 2.6 db/km. 3 others however show an almost equal tendency to decrease, average 2.0 db/km. The remaining 24 cases show no obvious trend towards increase or decrease of rainfall with height, and it is not unreasonable to conclude that the variation in the 8 cases was due to slightly heavier or lighter rain drifting into the beam in the course of the scan. Mean values of signal intensity for the whole 32 scans have been plotted (Fig.21, full curve). There is seen to be remarkably little change of intensity with height below the freezing level.

Detailed records of cloud and humidity are not available for the observing site; the nearest hourly weather reports are those from Cranfield,

12 mi northwest of East Hill (Table 5). Complete coincidence in the occurrence of the steady rain cannot be expected at stations 12 mi apart, but these reports suggest very similar cloud conditions in each of the periods, with variable amounts of thin fractostratus, base 500 to 1,000 ft, and a main overcast cloud layer close to 2,000 ft in the steady rain. It will be seen that whenever the rain was continuous the relative humidity recorded lay between 93 and 99%. This suggests that evaporation of drops below cloud base was negligible, and could not mask any measurable coalescence effect. Indeed the upper air soundings at both Hemsby and Crawley were close to the saturated adiabatic, with humidities very close to saturation, to great heights.

Table 5. Hourly observations at Cranfield, 12 mi northwest of East Hill

Date	Time GMT	Cloud amounts and bases (ft) Scud	Main layer	R.H. %	Weather
5. 9.55	1000	5/8 St 500	8/8 Ns 4000	95	$cr_o r_o$
	1100	3/8 Fs 800	8/8 Sc 1500	95	cir_o
19.10.55	1000	3/8 Sc 1200	8/8 Ns 5000	88	ϕcir_o
	1100	7/8 Fs 800	8/8 Ns 1500	99	crr
20.10.55	1000	3/8 Fs 800	8/8 Sc 2000	95	cir_o
	1100	2/8 Fs 1000	8/8 Sc 2000	95	crr
	1200	4/8 Fs 900	8/8 Sc 2200	93	crr
	1300	3/8 Fs 1100	8/8 Sc 2000	94	$cr_o r_o$

ϕ Rain very light at Cranfield until 1006.

It is of interest to compare this measured signal intensity curve (Fig.21) with that derived from the step-by-step computation of accretion and coalescence effects by Mason and Ramanadham, which was for very similar conditions. The rainfall in our observations averaged 2.7 mm hr^{-1}, while they assumed a steady rate of rainfall at the melting level of 3 mm hr^{-1}, a 1 km fall through cloud of cloud-water content 0.2 gm m^{-3}, and a cloud base 1 km above the ground. They computed that the radar echo intensities just below the melting level, at the cloud base, and at the ground would be in the ratios 1 : 1.86 : 2.2. In this computation 70% of the increase of intensity in cloud (1.9 db/km) was contributed by the accretion of cloud droplets, 30% (0.8 db/km) by the coalescence of drops. This rate of growth, applied at point A just below the melting level in Fig. 21 is represented by the broken line. It represents growth by accretion of cloud droplets and by coalescence of drops above cloud base, and by coalescence of drops alone below cloud base with negligible evaporation, for which there is strong support from the Cranfield observations of Table 5.

Very similar calculations have been made by Rigby, Marshall and Hitschfeld (13). For a slightly higher rainfall of 4.5 mm hr^{-1} and an assumed cloud-water content 0.4 gm m^{-3} they computed a rate of increase of intensity of 2.4 db/km due to accretion of droplets, and 0.8 db/km due to coalescence of drops. The rather small difference from Mason and Ramanadham's result despite the greater cloud density assumed is due to their taking a somewhat different initial distribution of drop sizes at the melting level.

There is seen to be a clear separation between the computed and measured growth curves in Fig. 21. The relative constancy of the measured signal intensity with height suggests that there was considerable uniformity in drop-size distribution once melting was complete and terminal velocities had been achieved, and that all significant growth of precipitation was occurring at and above the melting level. It seems that in these conditions the radar measurements at say 2,000 ft are an accurate measure of the rainfall at the ground assuming, as we think is reasonable, that no significant change occurs in the lowest 500 ft, which could not be investigated by radar.

Our failure to find any measurable increase of intensity which could be

attributed to accretion growth suggests that the cloud-water content below the freezing level was much lower than 0.2 gm m^{-3}. It may well be that the initial rain at a warm front washes out most of the cloud droplets below the melting level, and that, unless there is some cloud renewal mechanism at these low levels, e.g., orographic or frontal uplift, the cloud-water content thereafter remains low. One would then be forced to assume that the cloud at around 2,000 ft was merely a second layer of thin turbulence cloud of negligible water content. This could probably only be reliably confirmed by coincident aircraft investigations, but it may prove to be typical of warm-front precipitation in those parts of the British Isles in which orographic effects are unimportant.

The computed increase of 0.8 db/km due to coalescence of drops is necessarily more difficult to confirm experimentally. It was not in fact detected, and the measured values of Fig. 21 are so nearly constant below the melting level as to provide some evidence that the contribution to growth from such collisions does at least not exceed the computed values.

12. References

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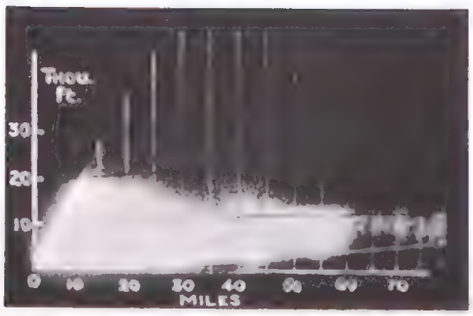

Fig. 1 0845 GMT 5th September 1955. Range-height photograph looking along the belt of precipitation. Unusually uniform echo.

Fig. 2 0929 GMT 5th September 1955. Plan-position photograph on 120 mi. range (range circles in 10 mi steps) shows belt of precipitation over the station 20 to 40 mi in width.

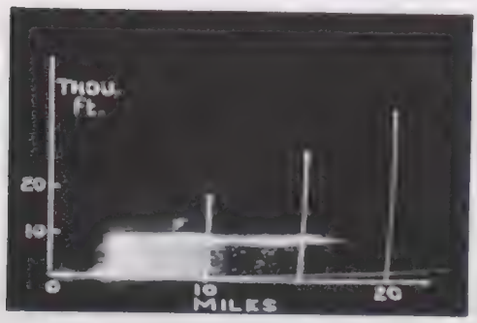

Fig. 3 0943 GMT 5th September 1955. Reduced gain photograph showing the melting level at 8,500 ft.

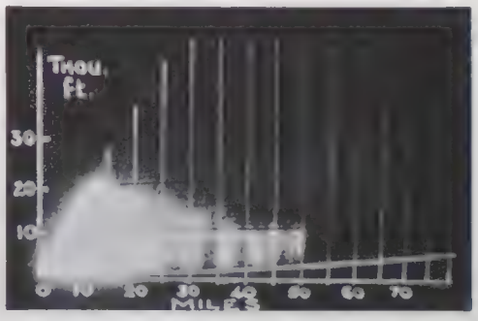

Fig. 4 0945 GMT 5th September 1955. Cellular structure has developed. Compare with much more uniform echo of fig. 1 .

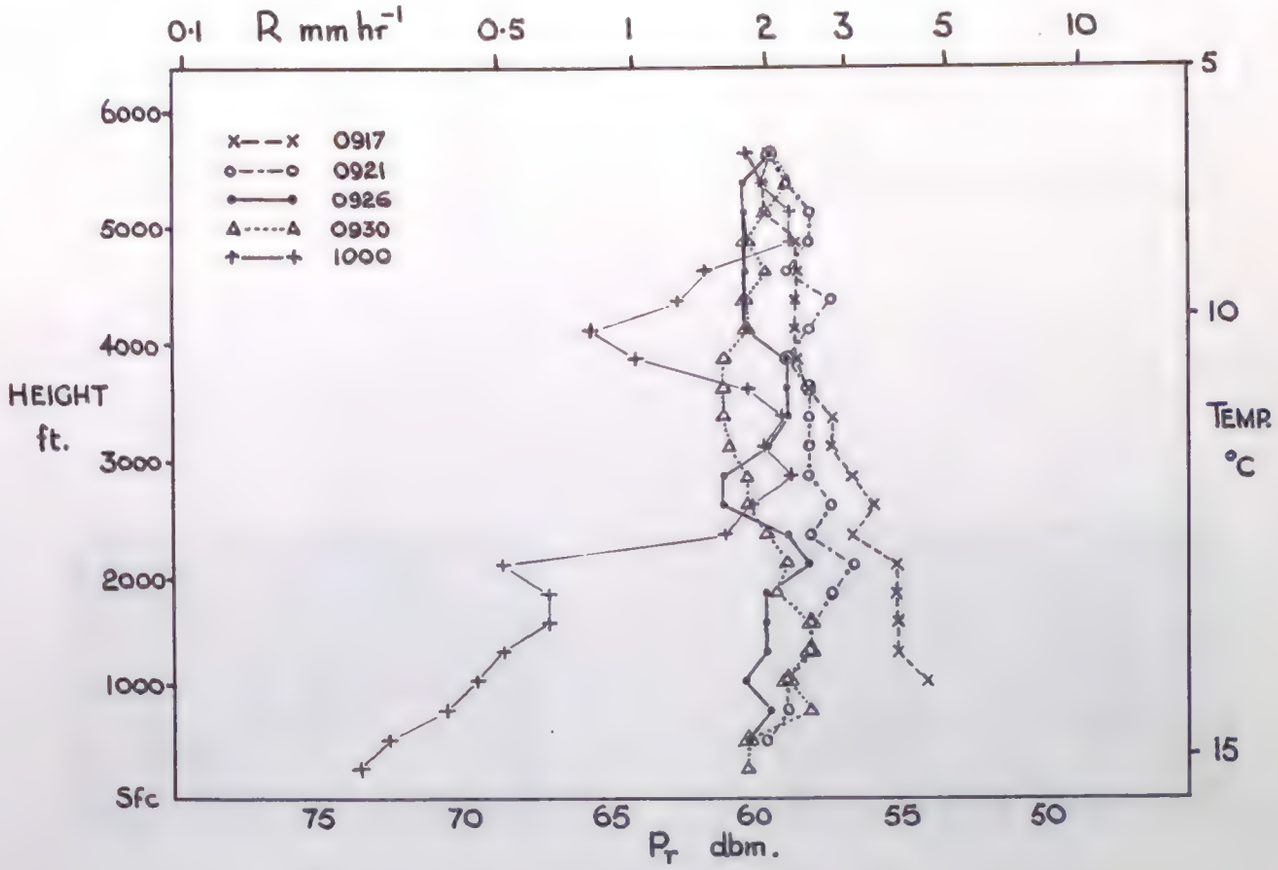

FIG. 5. SIGNAL INTENSITY MEASUREMENTS IN dbm (DECIBELS BELOW 1 MILLIWATT) ON 5TH SEPTEMBER 1955. RAIN UNSTEADY FOR LAST SCAN, AND HAD ALMOST CEASED AT LOWEST ELEVATIONS.

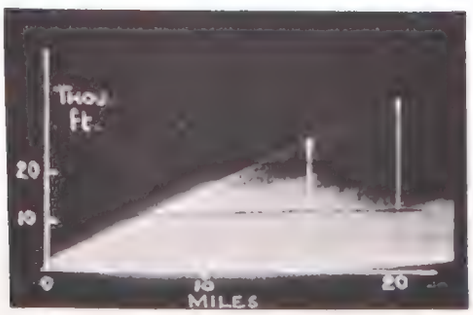

Fig. 6 1005 GMT 19th October 1955. An open scale range-height photograph of the precipitation within 20 mi. Very uniform echo.

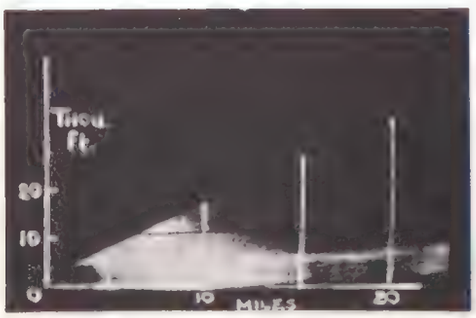

Fig. 7 1005 GMT 19th October 1955. Reduced gain photograph showing the bright band at the melting level at 5,000 ft.

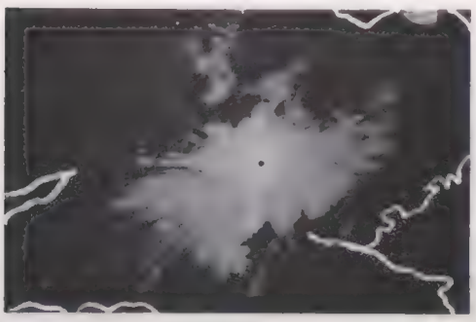

Fig. 8 1020 GMT 19th October 1955. Plan-position photograph on 60 mi range. (range circles in 5 mi steps). The lack of brightness is not caused by weakness of echo, but by a slight photographic fault.

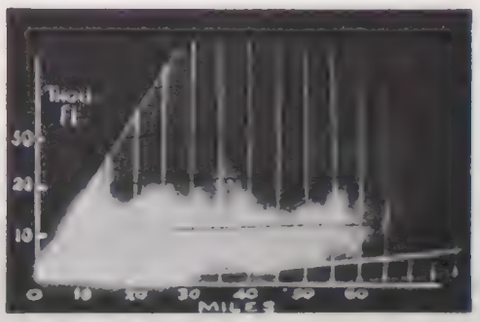

Fig. 9 1021 GMT 19th October 1955. Range-height photograph. Precipitation is probably more variable beyond 20 mi as evidenced by uneven tops.

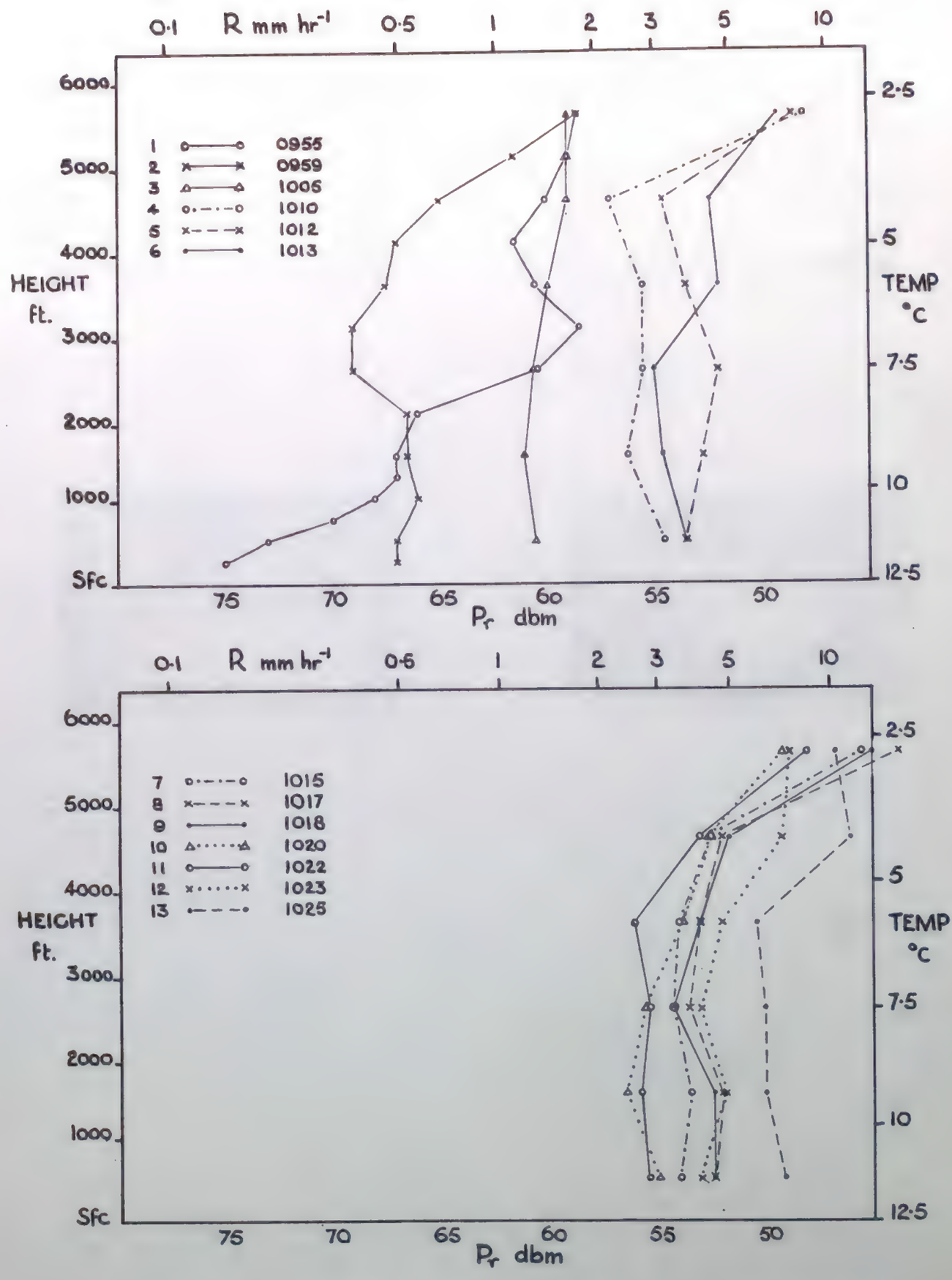

FIG. 10. SIGNAL INTENSITY MEASUREMENTS ON 19TH OCTOBER 1955. RAIN LIGHT AND VARIABLE UNTIL 1005. THE SCALE OF RATE OF RAINFALL APPLIES ONLY TO THE RAIN BELOW THE MELTING LAYER.

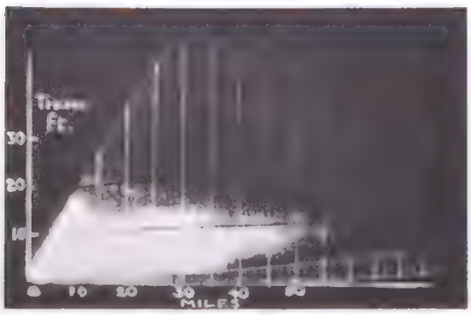

Fig. 11 0932 GMT 20th October 1955. Range-height photograph showing fairly uniform echo out to 60 mi, subject to range attenuation. The faint echo at 20,000 ft suggests ice crystal echo.

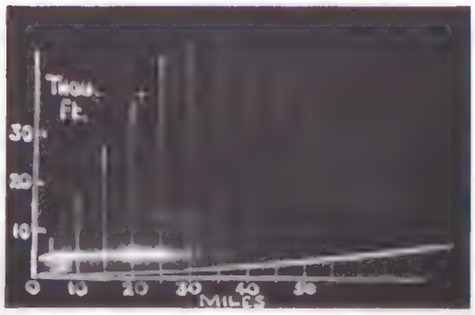

Fig. 12 0933 GMT 20th October 1955. Reduced gain photograph of the same precipitation showing uniform bright band at the melting level at 4,000 ft.

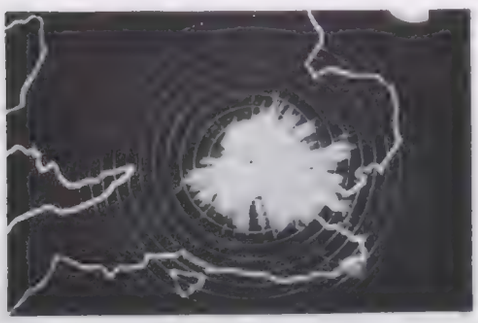

Fig. 13 1015 GMT 20th October 1955. Plan-position photograph on 120 mi range (range circles in 10 mi steps). Uniform echo to 30-60 mi.

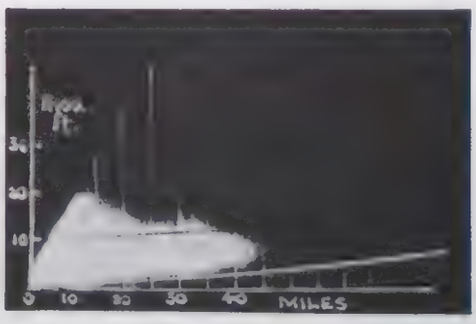

Fig. 14 1035 GMT 20th October 1955. Echo still uniform out to 45 mi.

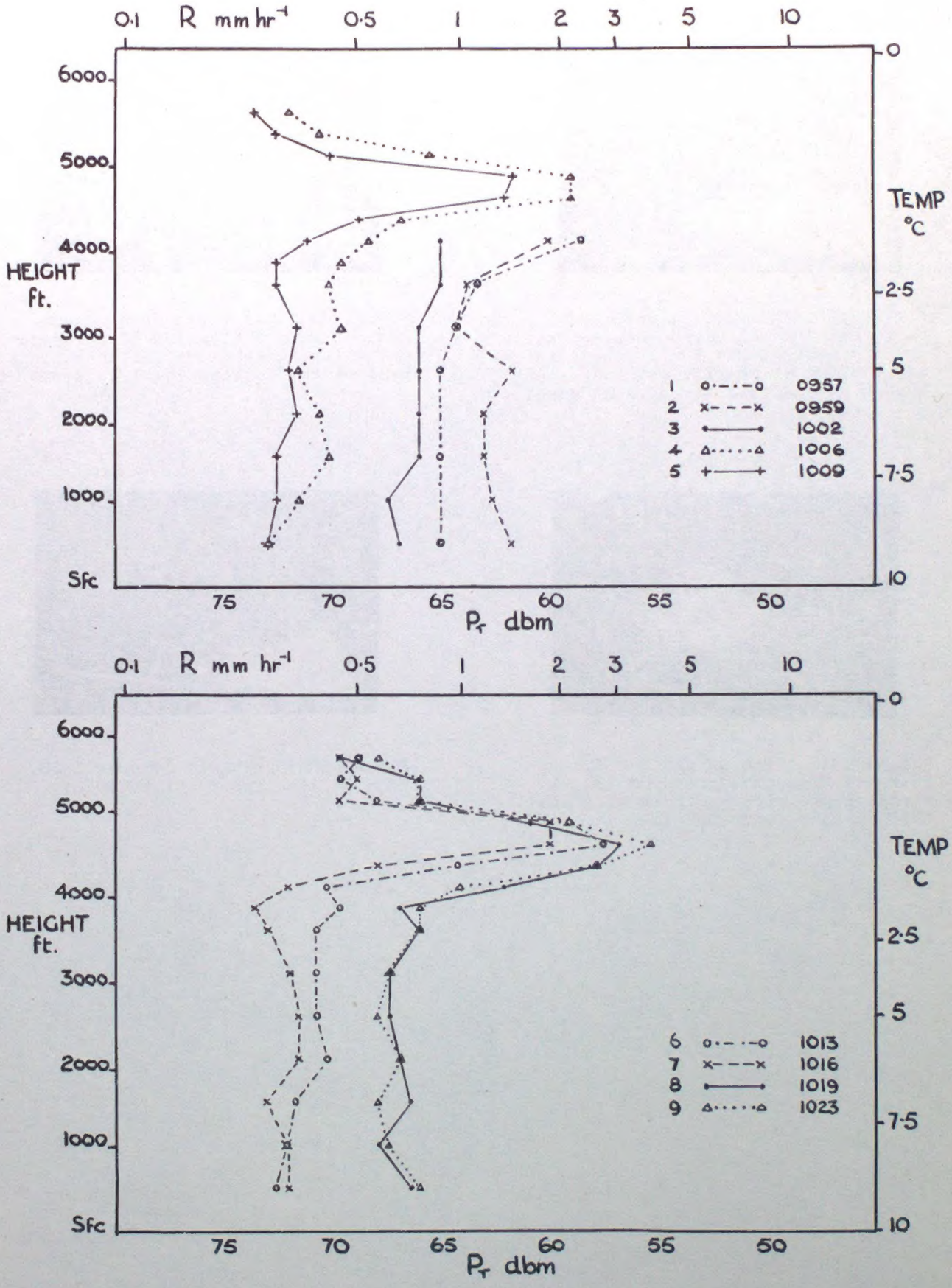

FIG. 15 SIGNAL INTENSITY MEASUREMENTS ON 20TH OCTOBER 1955. THE SCALE OF RATE OF RAINFALL APPLIES ONLY TO THE RAIN BELOW THE MELTING LAYER.

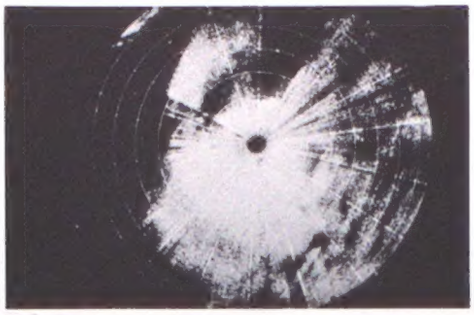

Fig.16 1156 GMT 20th October 1955. Plan-position photograph on 35 mi range (range circles in 5 mi steps) showing uniform precipitation over and to the South of East Hill.

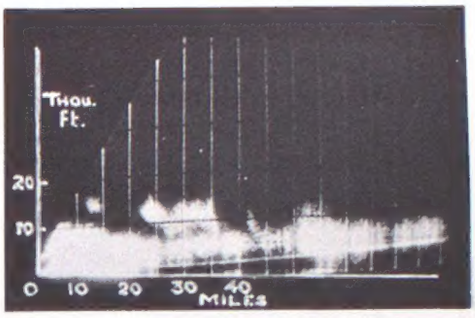

Fig.17 1212 GMT 20th October 1955. Range-height photograph looking SE showing uniform echo attenuating with range out to 20 mi, but much more variable beyond.

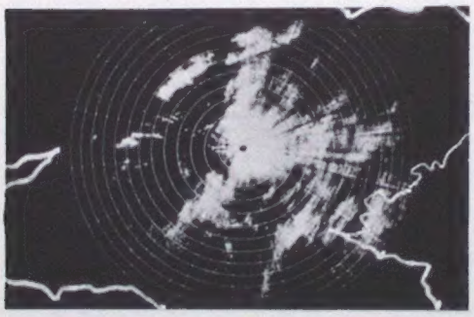

Fig.18 1230 GMT 20th October 1955. Plan-position photograph on 70 mi range (range circles in 5 mi steps) showing rain area of fig.16 almost past East Hill.

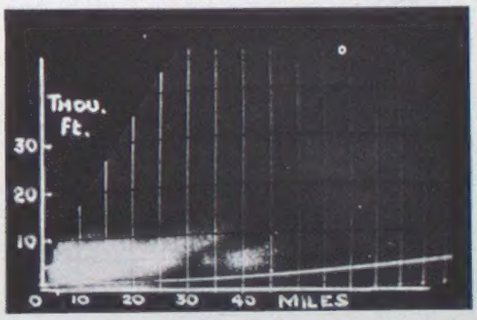

Fig.19 1246 GMT 20th October 1955. Range-height photograph looking NE through the rain, which had by then cleared East Hill.

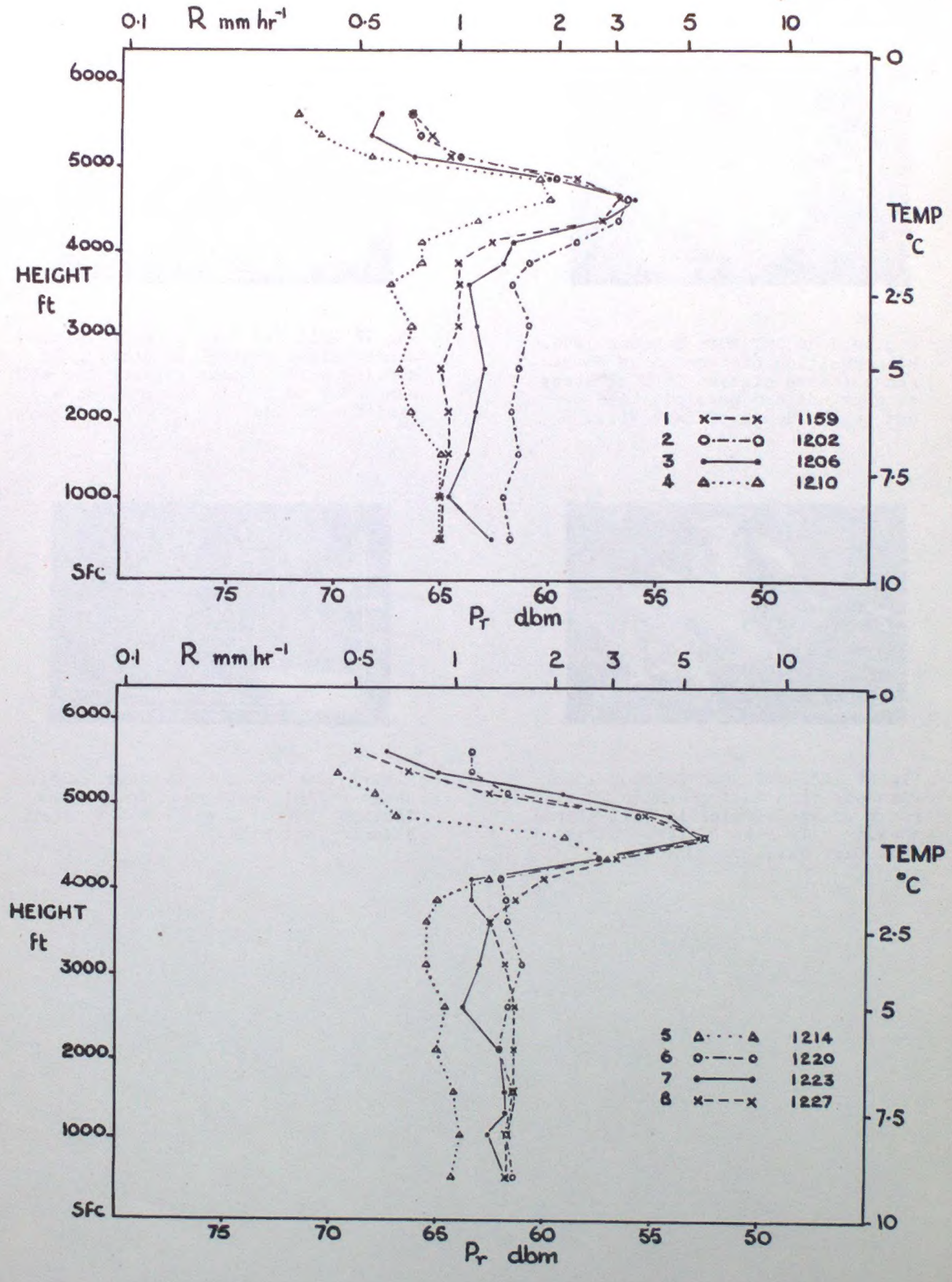

FIG. 20 SIGNAL INTENSITY MEASUREMENTS ON 20TH OCTOBER 1955. THE SCALE OF RATE OF RAINFALL APPLIES ONLY TO THE RAIN BELOW THE MELTING LAYER.

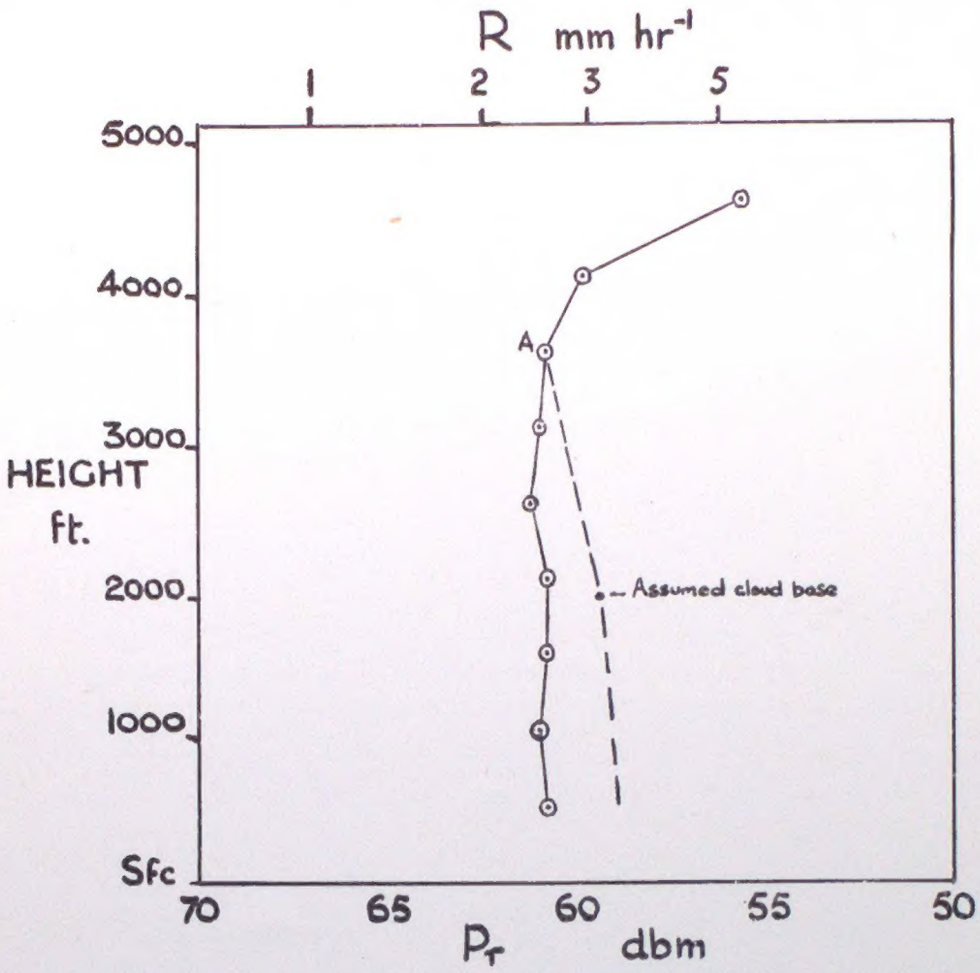

FIG.21 Mean signal intensity from 32 scans in steady rain. There is complete absence of any evidence of growth of drops below the melting layer. The broken line is a computed growth curve due to Mason and Ramanadham applied to the fall from the level marked A.